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Mohand Transform of Bessel's Functions

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Abstract: Bessel's functions are very useful for solving many equations in cylindrical or spherical coordinates such as heat equation, wave equation, Laplace equation, Helmholtz equation, Schrodinger equation. In this research, we find the Mohand transform of Bessel's functions.

Keywords: Mohand transform, Convolution theorem, Inverse Mohand transform, Bessel function.

1. INTRODUCTION:

In the advance time, Bessel's functions play a very important role for solving the problems of mathematical physics, atomic physics, acoustics, radio physics, nuclear physics, engineering and sciences such as flux distribution in a nuclear reactor, fluid mechanics, heat transfer, vibrations, hydrodynamics, stress analysis etc.

Bessel's function of order n, where n is the non-negative integer, is given by [1-5]

$$J_n(t) = \frac{t^n}{2^n n!}\left[1 - \frac{t^2}{2.(2n+2)} + \frac{t^4}{2.4.(2n+2)(2n+4)} - \frac{t^6}{2.4.6.(2n+2)(2n+4)(2n+6)} + \cdots..\right] .....(1)$$

For $n = 0$, we have Bessel's function of zero order denoted by $J_0(t)$ and it is defined by the following infinite power series as

$$J_0(t) = 1 - \frac{t^2}{2^2} + \frac{t^4}{2^2.4^2} - \frac{t^6}{2^2.4^2.6^2} + \cdots..(2)$$

For $n = 1$, we have Bessel's function of order one denoted by $J_1(t)$ and it is defined by the following infinite power series as

$$J_1(t) = \frac{t}{2} - \frac{t^3}{2^2.4} + \frac{t^5}{2^2.4^2.6} - \frac{t^7}{2^2.4^2.6^2.8} + ..(3)$$

Equation (3) can be written as

$$J_1(t) = \frac{t}{2} - \frac{t^3}{2^3.2!} + \frac{t^5}{2^5.2!.3!} - \frac{t^7}{2^7.3!.4!} + \cdots.(4)$$

For $n = 2$, we have Bessel's function of order two denoted by $J_2(t)$ and it is defined by the following infinite power series as

$$J_2(t) = \frac{t^2}{2.4} - \frac{t^4}{2^2.4.6} + \frac{t^6}{2^2.4^2.6.8} - \frac{t^8}{2^2.4^2.6^2.8.10} + \cdots..(5)$$

Mohand and Mahgoub [7] defined the new integral transform "Mohand transform'' of the function $F(t)$ for $t \geq 0$ as

$$M\{F(t)\} = v^2 \int_0^\infty F(t)e^{-vt}dt$$

$$= R(v), k_1 \leq v \leq k_2 \ldots\ldots\ldots\ldots.(6)$$

where M is Mohand transform operator.

If the function $F(t)$ for $t \geq 0$ is piecewise continuous and of exponential order then Mohand transform of the function $F(t)$ for $t \geq 0$ exists. These conditions are sufficient conditions for the existence of Mohand transform of the function $F(t)$ for $t \geq 0$

Kumar et al. [8] used Mohand transform for solving linear Volterra integro-differential equations. Aggarwal et al. [9] defined Aboodh transform of Bessel's functions. Aggarwal [10] gave the Elzaki transform of Bessel's functions. Kamal transform of Bessel's functions was given by Aggarwal [11]. Aggarwal et al. [12] obtained Mahgoub transform of Bessel's functions.

The goal of the present research is to determine Mohand transform of Bessel's functions of order zero, one and two.

2. SOME USEFUL PROPERTIES OF MOHAND TRANSFORM:

2.1 Linearity property:

If $M\{F_1(t)\} = R_1(v)$ and $M\{F_2(t)\} = R_2(v)$ then

$$M\{aF_1(t) + bF_2(t)\} = aM\{F_1(t)\} + bM\{F_2(t)\}$$

$= aR_1(v) + bR_2(v)$,where a, b are arbitrary constants.

Proof: By the definition of Mohand transform, we have

$$M\{F(t)\} = v^2 \int_0^\infty F(t)e^{-vt}dt$$

$$\Rightarrow M\{aF_1(t) + bF_2(t)\} = v^2 \int_0^\infty [aF_1(t) + bF_2(t)]e^{-vt}dt$$

$$\Rightarrow M\{aF_1(t) + bF_2(t)\} = av^2 \int_0^\infty F_1(t)e^{-vt}dt + bv^2 \int_0^\infty F_2(t)e^{-vt}dt$$

$$\Rightarrow M\{aF_1(t) + bF_2(t)\} = aM\{F_1(t)\} + bM\{F_2(t)\}$$

$$\Rightarrow M\{aF_1(t) + bF_2(t)\} = aR_1(v) + bR_2(v),$$

where a, b are arbitrary constants.

2.2 Change of scale property:

If $M\{F(t)\} = R(v)$ then

$$M\{F(at)\} = aR\left(\frac{v}{a}\right)$$

Proof: By the definition of Mohand transform, we have

$$M\{F(at)\} = v^2 \int_0^\infty F(at)e^{-vt}dt \dots\dots\dots (7)$$

Put $at = p \Rightarrow adt = dp$ in equation (7), we have

$$M\{F(at)\} = \frac{v^2}{a} \int_0^\infty F(p)e^{\frac{-vp}{a}}dp$$

$$\Rightarrow M\{F(at)\} = a\left[\frac{v^2}{a^2} \int_0^\infty F(p)e^{\frac{-vp}{a}}dp\right]$$

$$\Rightarrow M\{F(at)\} = aR\left(\frac{v}{a}\right) \dots\dots\dots\dots\dots (8)$$

2.3 Mohand transform of the derivatives of the function $F(t)$ [7-8]:

If $M\{F(t)\} = R(v)$ then

a) $M\{F'(t)\} = vR(v) - v^2F(0)$
b) $M\{F''(t)\} = v^2R(v) - v^3F(0) - v^2F'(0)$
c) $M\{F^{(n)}(t)\} = v^nR(v) - v^{n+1}F(0) - v^nF'(0) - \cdots \dots - v^2F^{(n-1)}(0)$

2.4 Mohand transform of some basic mathematical functions [7, 8]:

Table: 1

S.N.	$F(t)$	$M\{F(t)\} = R(v)$
1.	1	v
2.	t	1
3.	t^2	$\frac{2!}{v}$
4.	$t^n, n \in N$	$\frac{n!}{v^{n-1}}$
5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{v^{n-1}}$
6.	e^{at}	$\frac{v^2}{v-a}$
7.	$sinat$	$\frac{av^2}{(v^2+a^2)}$
8.	$cosat$	$\frac{v^3}{(v^2+a^2)}$
9.	$sinhat$	$\frac{av^2}{(v^2-a^2)}$
10.	$coshat$	$\frac{v^3}{(v^2-a^2)}$

3. Relation between $J_0(t)$ and $J_1(t)$[9, 12]:

$$\frac{d}{dt}J_0(t) = -J_1(t) \dots\dots\dots. (9)$$

4. Relation between $J_0(t)$ and $J_2(t)$[9-12]:

$$J_2(t) = J_0(t) + 2J_0''(t) \dots\dots\dots (10)$$

5. Mohand transform of Bessel's functions:

5.1 Mohand transform of Bessel's function of zero order :

Applying Mohand transform both sides on equation(2), we get

$$M\{J_0(t)\} = M\{1\} - \frac{1}{2^2}M\{t^2\} + \frac{1}{2^2.4^2}M\{t^4\} - \frac{1}{2^2.4^2.6^2}M\{t^6\} + \cdots ..$$

$$= v - \frac{1}{2^2}\left(\frac{2!}{v}\right) + \frac{1}{2^2.4^2}\left(\frac{4!}{v^3}\right) - \frac{1}{2^2.4^2.6^2}\left(\frac{6!}{v^5}\right) + \cdots \dots ....$$

$$= \left[v - \frac{1}{2}\left(\frac{1}{v}\right) + \frac{1.3}{2.4}\left(\frac{1}{v^3}\right) - \frac{1.3.5}{2.4.6}\left(\frac{1}{v^5}\right) + \cdots \dots \dots \dots .\right]$$

$$= v\left[1 - \frac{1}{2}\left(\frac{1}{v}\right)^2 + \frac{1.3}{2.4}\left(\frac{1}{v^2}\right)^2 - \frac{1.3.5}{2.4.6}\left(\frac{1}{v^3}\right)^2 + \cdots \dots \dots\right]$$

$$= v\left(1 + \frac{1}{v^2}\right)^{-1/2} = \frac{v^2}{\sqrt{(1+v^2)}} \dots \dots ..(11)$$

5.2 Mohand transform of Bessel's function of order one :

Applying Mohand transform both sides on equation (9), we have

$$M\{J_1(t)\} = -M\{J_0{}'(t)\} \dots \dots \dots \dots \dots .(12)$$

Now using the property, Mohand transform of derivative of the function, on equation (12), we have

$$M\{J_1(t)\} = -[vM\{J_0(t)\} - v^2 J_0(0)]..(13)$$

Using equations (2) and (11) in equation (13), we have

$$M\{J_1(t)\} = -\left[v.\frac{v^2}{\sqrt{(1+v^2)}} - v^2\right]$$

$$M\{J_1(t)\} = v^2 - \frac{v^3}{\sqrt{(1+v^2)}} \dots \dots \dots .(14)$$

5.3 Mohand transform of Bessel's function of order two :

Applying Mohand transform both sides on equation (10), we have

$$M\{J_2(t)\} = M\{J_0(t)\} + 2M\{J_0{}''(t)\} \dots ..(15)$$

Now using the property, Mohand transform of derivative of the function and equation (11), in equation (15), we have

$$M\{J_2(t)\} = \frac{v^2}{\sqrt{(1+v^2)}} + 2\left[v^2 M\{J_0(t)\} - v^3 J_0(0) - v^2 J_0{}'(0)\right] \dots \dots .(16)$$

Using equations (2), (9) and (11) in equation (13), we have

$$M\{J_2(t)\} = \frac{v^2}{\sqrt{(1+v^2)}} + 2\left[v^2.\frac{v^2}{\sqrt{(1+v^2)}} - v^3 + v^2 J_1(0)\right] \dots \dots .(17)$$

Now using equation (3) in equation (17), we have

$$M\{J_2(t)\} = \frac{v^2}{\sqrt{(1+v^2)}} + \frac{2v^4}{\sqrt{(1+v^2)}} - 2v^3$$

$$= \frac{v^2 + 2v^4 - 2v^3\sqrt{(1+v^2)}}{\sqrt{(1+v^2)}} \dots \dots ..(18)$$

5.4 Mohand transform of $J_0(at)$:

From equation (11), we have

$$M\{J_0(t)\} = \frac{v^2}{\sqrt{(1+v^2)}}$$

Now using change of scale property of Mohand transform, we have

$$M\{J_0(at)\} = a\left[\frac{(v/a)^2}{\sqrt{(1+(v/a)^2)}}\right]$$

$$= \left[\frac{v^2}{\sqrt{(a^2+v^2)}}\right] \dots \dots \dots \dots ..(19)$$

5.5 Mohand transform of $J_1(at)$:

From equation (14), we have

$$M\{J_1(t)\} = v^2 - \frac{v^3}{\sqrt{(1+v^2)}}$$

Now using change of scale property of Mohand transform, we have

$$M\{J_1(at)\} = a\left[({}^{v}/_{a})^2 - \frac{({}^{v}/_{a})^3}{\sqrt{(1 + ({}^{v}/_{a})^2)}}\right]$$

$$= \frac{v^2}{a}\left[1 - \frac{v}{\sqrt{(a^2 + v^2)}}\right] \ldots\ldots\ldots\ldots\ldots (20)$$

5.6 Mohand transform of $J_2(at)$:

From equation (18), we have

$$M\{J_2(t)\} = \frac{v^2 + 2v^4 - 2v^3\sqrt{(1 + v^2)}}{\sqrt{(1 + v^2)}}$$

Now using change of scale property of Mohand transform, we have

$$M\{J_2(at)\} = a\left[\frac{({}^{v}/_{a})^2 + 2({}^{v}/_{a})^4 - 2({}^{v}/_{a})^3\sqrt{(1 + ({}^{v}/_{a})^2)}}{\sqrt{(1 + ({}^{v}/_{a})^2)}}\right]$$

$$= \frac{v^2}{a^2}\left[\frac{a^2 + 2v^2 - 2v\sqrt{(a^2 + v^2)}}{\sqrt{(a^2 + v^2)}}\right] \ldots\ldots\ldots\ldots (21)$$

6. CONCLUSIONS

In this paper, we have successfully discussed the Mohand transform of Bessel's functions of zero, one and two orders. In future, Mahgoub transform can be applied for solving Bessel's differential equations.

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